Section 7.1

Solve:

$$x^2 = 64 \qquad\qquad x^2 = 64$$
$$x^2 - 64 = 0 \qquad\qquad \sqrt{x^2} = \sqrt{64}$$
$$(x-8)(x+8) = 0 \quad \text{or} \quad x = \pm 8$$
$$x = 8 \quad x = -8$$
$$x \in \{8, -8\}$$

In the second line of the second version, we "took the square root" of two items. This was an operation.

I will explain the reason this "operation" results in two answers in a moment.

Consider: "Find the square root of 64" This is written: $x = \sqrt{64}$.

In this case there is ONLY one answer and that answer is 8. Writing $\sqrt{64}$ is just another way to write an 8….. as $\frac{12}{2}$ is another way to write a 6. We say that $x = \sqrt{64}$ asks for the "principle square root". If we wanted the – 8 answer, we would have to specifically ask for the negative root.

The reason we get two answer as in our first example is from the definition of absolute value:

$$\sqrt{\bigstar^2} = |\bigstar| = \begin{cases} -\bigstar & \text{if } \bigstar < 0 \\ \bigstar & \text{if } \bigstar \geq 0 \end{cases}$$

Actually we get two answers for $\sqrt{x^2}$, we get $+x$ and $-x$ and the square root of 64 is 8. Thus we have x = 8 and $-x = 8$ and end up with two solutions.

The rule is….

"If YOU put in the square root, then there are 2 answers."

"If the square root is already there, there one answer."

Section 7.2 Rational Exponents

This extends our definition of exponents. No longer can we "count items" to determine the exponent as we have before:

$$\left(x^3\right)^2$$
$$x^3 \cdot x^3$$
$$\underbrace{x \cdot x \cdot x \quad x \cdot x \cdot x}_{x^6}$$

Exponents can be rational numbers (fractions) instead of integers.

$$x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$$

$27^{\frac{2}{3}}$	$25^{\frac{3}{2}}$	$\sqrt[5]{t^{20}}$	$\sqrt{\sqrt[3]{x}}$
$\left(3^3\right)^{\frac{2}{3}}$	$\left(5^2\right)^{\frac{3}{2}}$	$\left(t^{20}\right)^{\frac{1}{5}}$	$\left(x^{\frac{1}{3}}\right)^{\frac{1}{2}}$
3^2	5^3		
9	125	t^4	$x^{\frac{1}{6}}$

All or our rules for exponents still apply.

When you write a square root symbol, be sure it covers everything that is inside the square root.

We want this: $\sqrt{13ab}$ rather than $\sqrt{13a}\ b$

Section 7.3 Multiplying Radical Expressions

$$a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = (a \cdot b)^{\frac{1}{n}} = \sqrt[n]{ab}$$

$\sqrt{500}$	$\sqrt{15} \cdot \sqrt{6}$
$\sqrt{100}\sqrt{5}$	$\sqrt{3 \cdot 5 \cdot 3 \cdot 2}$
$10\sqrt{5}$	$3\sqrt{10}$

For a square root, it takes 2 identical factors on the inside to get one factor on the outside.

In the square root of 100, we had two 10s so we got a single 10 on the outside. In the second example, we had a pair of 3s so we got ONE 3 on the outside.

If we had cube roots, we would need 3 identical factors on the inside to get ONE factor on the outside.